Kondo and Dicke effect in quantum-dots side coupled to a quantum wire

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Electron tunneling through quantum-dots side coupled to a quantum wire, in equilibrium and nonequilibrium Kondo regime, is studied. The mean-field finite-U slave-boson formalism is used to obtain the solution of the problem. We have found that the transmission spectrum shows a structure with two anti-resonances localized at the renormalized energies of the quantum dots. The DOS of the system shows that when the Kondo correlations are dominant there are two Kondo regimes with its own Kondo temperature. The above behavior of the DOS can be explained by quantum interference in the transmission through the two different resonance states of the quantum dots coupled to common leads. This result is analogous to the Dicke effect in optics. We investigate the many body Kondo states as a function of the parameters of the system.

I. INTRODUCTION

The Kondo effect in quantum dots (QDs) has been extensively studied in the last years[1,2,3]. The QDs allow studying systematically the quantum-coherence many-body Kondo state, due to the possibility of continuous tuning the relevant parameters governing the properties of this state, in equilibrium and nonequilibrium situations. Recently Kondo effect has been studied in side attach quantum dot[4] and parallel quantum dots[9,10]. Recent electron transport experiments showed that Kondo and Fano resonances occur simultaneously[5]. Multiple scattering of traveling electronic waves on a localized magnetic state are crucial for the formation of both resonances. The condition for the Fano resonance is the existence of two scattering channels: a discrete level and a broad continuum band[6].

An alternative configuration consists of two single QDs side attached to a perfect quantum wire (QW). This structure is reminiscent of the cross-bar-shaped quantum wave guides[7]. In this case, the QDs act as scattering centers in close analogy with the traditional Kondo effect[8]. This configuration was study previously by Stefański[11] and Tamura et at.[12].

In this work we study the transport properties of two single quantum dots side coupled to a quantum wire in the Kondo regime. We use the finite-U slave boson mean-field approach, which was initially developed by Kotliar and Ruckenstein[13] and used later by Bing Dong and X. L. Lei to study the transport through coupled double quantum dots connected to leads[14].This approach enforces the correspondence between the impurity fermions and the auxiliary bosons to a mean-field level to release the $U = \infty$ restriction. In quantum dots, this approach allows to treat the dot-lead coupling nonperturbatively for an arbitrary strength of the Coulomb interaction U[14]. We have found that the transmission spectrum shows a structure with two anti-resonances localized at the renormalized energies of the quantum dots. The DOS of the system shows that when the Kondo correlations are dominant there are two Kondo regimes each with its own Kondo temperature. The above behavior of the DOS can be explained by quantum interference in the transmission through the two different resonance states of the quantum dots coupled to common lead. This phenomenon is in analogy to the Dicke effect in quantum optics, that takes place in the spontaneous emission of two closely-lying atoms radiating a photon into the same environment[16]. In the electronic case, however, the decay rates (level broadening) are produced by the indirect coupling of the up-down QDs, giving rise to a fast (*superradiant*) and a slow (*subradiant*) mode. Recently, Brandes reviewed the Dicke effect in mesoscopic systems[17].

II. MODEL

Let us consider two single quantum dot (2QD) side coupled to a perfect quantum wire (QW) (see Fig. 1). We adopt the two-impurities Anderson Hamiltonian. Each dot has a single level energy ε_l (with $l = 1, 2$), and a intra-dot Coulomb repulsion U. The two side attached quantum-dots are coupled to the QW with coupling t_0. The QW sites have local energies $\varepsilon_{wi,\sigma} = 0$ and a hopping parameter t.

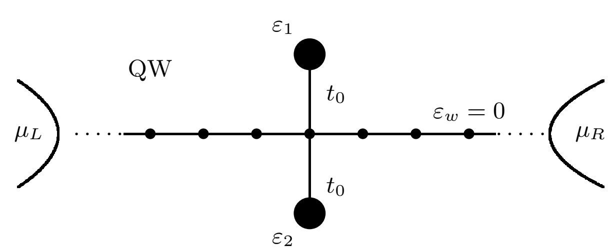

FIG. 1: Scheme of side-coupled quantum dots attached laterally to a perfect quantum wire (QW). The QW is coupled to the left (L) and right (R) noninteracting leads.

The corresponding Hamiltonian model is,

$$H_0 = -t\sum_{i,\sigma}\left(c^\dagger_{i,\sigma}c_{i+1,\sigma} + \text{H.c.}\right) + \sum_{l=1,2,\sigma}\left[-t_{l,\sigma}\left(c^\dagger_{0,\sigma}f_{l,\sigma} + \text{H.c.}\right) + \left(\varepsilon_{l,\sigma} + \frac{U}{2}\hat{n}_{l,-\sigma}\right)\hat{n}_{l,\sigma}\right] \tag{1}$$

where $c^\dagger_{i,\sigma}$ ($c_{i,\sigma}$) is the creation (annihilation) operator of an electron with spin σ at the i-th site of the quantum wire; $f^\dagger_{l,\sigma}$ ($f_{l,\sigma}$) is the creation (annihilation) operator of an electron with spin σ in the l-th QD and $\hat{n}_{l,\sigma}$ is the corresponding number operator.

To find the solution of this correlated fermions system, we appeal to an analytical approach where, generalizing the infinite-U slave-boson approximation[18] the Hilbert space is enlarged at each site, to contain in addition to the original fermions a set of four bosons[13] represented by the creation (annihilation) operators $e^\dagger_l$ (e_l), $p^\dagger_{l,\sigma}$ ($p_{l,\sigma}$), and $d^\dagger_l$ (d_l) for the l-th dot.They act as projectors onto empty, single occupied (with spin up and down) and doubly occupied electron states, respectively. Then, each creation (annihilation) operator of an electron with spin σ in the l-th QD, is substituted by $f^\dagger_{l,\sigma}\tilde{Z}^\dagger_{l,\sigma}$ ($\tilde{Z}_{l,\sigma}f_{l,\sigma}$) where:

$$\tilde{Z}_{l,\sigma} = \frac{e^\dagger_l p_{l,\sigma} + p^\dagger_{l,-\sigma}d_l}{\sqrt{1 - d^\dagger_l d_l - p^\dagger_{l,\sigma}p_{l,\sigma}}\sqrt{1 - e^\dagger_l e_l - p^\dagger_{l,-\sigma}p_{l,-\sigma}}} \tag{2}$$

The denominator is chosen to reproduce the correct $U \to 0$ limit in the mean-field approximation without changing neither the eigenvalues nor the eigenvector.

The constraint, i.e., the completeness relation $\sum_\sigma p^\dagger_{l,\sigma}p_{l,\sigma} + b^\dagger_l b_l + d^\dagger_l d_l = 1$ and the condition among fermions and bosons $n_{l,\sigma} - p^\dagger_{l,\sigma}p_{l,\sigma} - d^\dagger_l d_l = 0$, will be incorporated with Lagrange multipliers $\lambda^{(1)}_l$ and $\lambda^{(2)}_{l,\sigma}$ into the Hamiltonian. Also in the mean-field approximation all the boson operators are replaced by their expectation value which can be chosen, without loss of generality, as real numbers.

The Hamiltonian in this new and enlarged Hilbert space, is, $\mathcal{H} = H_b + H_e$, where

$$\begin{aligned} H_b &= \sum_{l=1,2}\lambda^{(1)}_l\left(p^2_{l,\uparrow} + p^2_{l,\downarrow} + e^2_l + d^2_l - 1\right) \\ &\quad - \sum_{l=1,2,\sigma}\lambda^{(2)}_{l,\sigma}\left(p^2_{l,\sigma} + d^2_l\right) + U\sum_{l=1,2}d^2_l\,, \end{aligned} \tag{3}$$

depends explicitly only upon the boson expectation values and the Lagrange multipliers, and

$$\begin{aligned} H_e &= -t\sum_{i,\sigma}\left(c^\dagger_{i,\sigma}c_{i+1,\sigma} + \text{H.c.}\right) \\ &\quad + \sum_{l=1,2,\sigma}\left[-\tilde{t}_{l,\sigma}\left(c^\dagger_{0,\sigma}f_{l,\sigma} + \text{H.c.}\right) + \tilde{\varepsilon}_{l,\sigma}n_{l,\sigma}\right] \end{aligned} \tag{4}$$

is a tight-binding Hamiltonian that depends implicitly on the boson expectation values through the parameters: $\tilde{\varepsilon}_{l,\sigma} = \varepsilon_{l,\sigma} + \lambda^{(2)}_{l,\sigma}$, $\tilde{t}_{l,\sigma} = t_0\langle\tilde{Z}_{l,\sigma}\rangle$.

As we work at zero temperature, the boson operators expectation values and the Lagrange multipliers are determined by minimizing the energy $\langle\mathcal{H}\rangle$ with respect to these quantities. It is obtained in this way, a set of non-linear equations for each quantum dot, relating the expectation values of the four bosonic operators, the three Lagrange multipliers and the electronic expectation values,

$$p^2_{l,\sigma} = \langle\hat{n}_{l,\sigma}\rangle - d^2_l\,, \tag{5a}$$

$$e^2_l = 1 - \sum_s\langle\hat{n}_{l,s}\rangle + d^2_l\,, \tag{5b}$$

$$\lambda^{(1)}_l = \frac{t_0}{e_l}\sum_s\langle f^\dagger_{l,s}c_{0,s}\rangle\frac{\partial\langle\tilde{Z}_{l,s}\rangle}{\partial e_l}\,, \tag{5c}$$

$$\lambda^{(1)}_l - \lambda^{(2)}_{l,\sigma} = \frac{t_0}{p_{l,\sigma}}\sum_s\langle f^\dagger_{l,s}c_{0,s}\rangle\frac{\partial\langle\tilde{Z}_{l,s}\rangle}{\partial p_{l,\sigma}}\,, \tag{5d}$$

$$U + \lambda^{(1)}_l - \sum_s\lambda^{(2)}_{l,s} = \frac{t_0}{d_l}\sum_s\langle f^\dagger_{l,s}c_{0,s}\rangle\frac{\partial\langle\tilde{Z}_{l,s}\rangle}{\partial d_l}\,. \tag{5e}$$

where l is the dot index, s, σ are spin indexes and $\langle\tilde{Z}_{l,s}\rangle$ satisfies,

$$\langle\tilde{Z}_{l,s}\rangle = \frac{p_{l,s}\left(e_l + d_l\right)}{\sqrt{\left(1 - d^2_l - p^2_{l,s}\right)\left(1 - e^2_l - p^2_{l,-s}\right)}}. \tag{6}$$

To obtain the electronic expectation values $\langle\cdots\rangle$, the Hamiltonian, H_e, is diagonalized and their stationary states can be written as

$$|\psi_k\rangle = \sum_{j=-\infty}^{\infty}a^k_j\,|j\rangle + \sum_{l=1}^{2}b^k_l\,|l\rangle\,, \tag{7}$$

where a^k_j and b^k_l are the probabilities amplitudes to find the electron at the site j and at the l-th QD respectively, with energy $\omega = -2t\cos k$. As we study the paramagnetic case the spin index is neglected.

The amplitudes a^k_j and b^k_l obey the following linear difference equations

$$\omega a_j^k = -t(a_{j+1}^k + a_{j-1}^k)\,, \quad j \neq 0\,, \tag{8a}$$
$$\omega a_0^k = -t(a_1^k + a_{-1}^k) - \tilde{t}_1 b_1^k - \tilde{t}_2 b_2^k\,, \tag{8b}$$
$$(\omega - \tilde{\varepsilon}_1) b_1^k = -\tilde{t}_1 a_0^k\,, \tag{8c}$$
$$(\omega - \tilde{\varepsilon}_2) b_2^k = -\tilde{t}_2 a_0^k\,. \tag{8d}$$

In order to study the solutions of Eqs. (8), we assume that the electrons are described by a unitary incident amplitude plane wave and reflection and transmission amplitudes r and τ, respectively. That is,

$$a_j^k = \mathrm{e}^{\mathrm{i}k\cdot j} + r\mathrm{e}^{-\mathrm{i}k\cdot j}\,;\ (k\cdot j < 0)\,, \tag{9a}$$
$$a_j^k = \tau \mathrm{e}^{\mathrm{i}k\cdot j} \qquad ; \qquad (k\cdot j > 0)\,. \tag{9b}$$

Inserting Eqs. (9) into Eqs. (8), we get an inhomogeneous system of linear equations for τ, r, a_j^k and b_l^k, leading to the following expression in equilibrium

$$\tau = \frac{1}{1 + i(\frac{\tilde{\Gamma}_1}{\omega - \tilde{\varepsilon}_1} + \frac{\tilde{\Gamma}_2}{\omega - \tilde{\varepsilon}_2})}\,, \tag{10}$$

where $\tilde{\Gamma}_l = \pi \tilde{t}_l^2 \rho_0(\omega)$ $(l = 1, 2)$ is the renormalized coupling between each quantum-dot and the leads of density of states $\rho_0(\omega)$. In spite of the apparent simplicity of the expression, it is necessary remember that the quantity $\tilde{t}_l$ implicitly depends on the expectation values of the boson operators also as fermion operators.

The transmission probability is given by $T = |\tau|^2$,

$$T(\omega) = \frac{1}{1 + (\frac{\tilde{\Gamma}_1}{\omega - \tilde{\varepsilon}_1} + \frac{\tilde{\Gamma}_2}{\omega - \tilde{\varepsilon}_2})^2}\,. \tag{11}$$

From the amplitudes b_1^k and b_2^k we obtain the local density of states (LDOS) at the quantum dot l (with $l = 1, 2$). In equilibrium that is,

$$\rho_l(\omega) = \frac{1}{\pi \tilde{\Gamma}_l} \frac{(\frac{\tilde{\Gamma}_l}{\omega - \tilde{\varepsilon}_l})^2}{1 + (\frac{\tilde{\Gamma}_1}{\omega - \tilde{\varepsilon}_1} + \frac{\tilde{\Gamma}_2}{\omega - \tilde{\varepsilon}_2})^2}\,. \tag{12}$$

In the nonequilibrium case, we suppose a finite source-drain biased with a symmetric voltage drop. The incident electrons from the left side (L), they are in equilibrium with thermodynamical potential $\mu_L = V/2$, and the incidents from the right side (R), they are in equilibrium with thermodynamical potential $\mu_R = -V/2$.

Once the amplitudes $a_{j,\sigma}^k$ and $b_{j,\sigma}^k$ are known, the electronic expectation values is obtained from,

$$\langle f_l^\dagger c_j \rangle = \frac{1}{2} \sum_{\alpha = L,R} \frac{1}{N} \sum_{k_\alpha} f\left(\epsilon_{k_\alpha} - \mu_\alpha\right) b_l^{k_\alpha *} a_j^{k_\alpha} \tag{13}$$

And the current is obtained from,

$$J = 2\frac{2e}{\hbar} t \sum_{\alpha, k_\alpha} f\left(\epsilon_{k_\alpha} - \mu_\alpha\right) \mathrm{Im}\{a_0^{k_\alpha *} a_1^{k_\alpha}\} \tag{14}$$

where $f\left(\epsilon_{k_\alpha} - \mu_\alpha\right)$ it is the Fermi function for incident electrons from the α side.

III. RESULTS

We solve numerically the set of nonlinear equations and take typical values for the parameters that define the system, $t = 25\,\Gamma$, $t_0 = 5\,\Gamma$ where $\Gamma = \pi t_0^2 \rho_0(0)$ is taken to be the unit of energy.

We consider first the situation in equilibrium where the two dots local state energies are set by $\varepsilon_1 = V_g - \delta V$, and $\varepsilon_2 = V_g + \delta V$. We choose the value of $V_g = -3\Gamma$. From now on all energies in units of Γ.

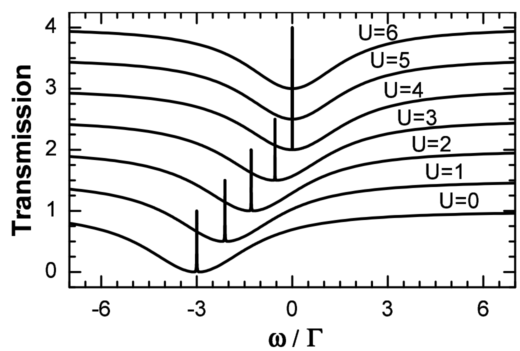

FIG. 2: Transmission spectrum in equilibrium for $V_g = -3$, $\delta V = 0.1$ and various values of U.

The transmission probability, T, is displayed in Fig. 2 for various values of U. The transmission probability always reaches zero at $\omega = \tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ and unitary value at $\omega = (\tilde{\varepsilon}_1 + \tilde{\varepsilon}_2)/2$. For small values of U the anti-ferromagnetic spin-spin correlation between the dots is dominant and the system does not posses a Kondo regime[15]. Increasing U, a sharp feature develops close to the Fermi energy ($\omega = 0$), indicating the appearance of a Kondo resonance.

For U sufficiently large the transmission can be written approximately as the superposition of a Fano and a Briet-Wigner line shapes,

$$T(\omega) \approx \frac{(\epsilon + q)^2}{\epsilon^2 + 1} + \frac{\tilde{\Delta}^2}{\omega^2 + \tilde{\Delta}^2}\,, \tag{15}$$

where $\epsilon = \omega/2\tilde{\Gamma}$, $q = 0$, with $\tilde{\Delta} = \delta\tilde{V}^2/2\tilde{\Gamma}$.

The DOS gives us more details about the formation of the Kondo resonance. The DOS is displayed in Fig. 3. In the Kondo regime the DOS can be written as the superposition of the two Lorentzian. These results imply

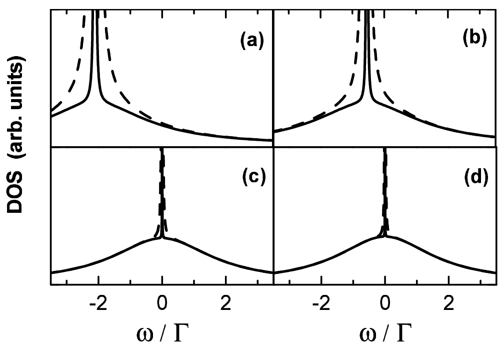

FIG. 3: DOS for $V_g = -3$, $\delta V = 0.1$ (solid line), 0.5 (dashed line). The on site energy U, is (a) 1, (b) 3, (c) 5 and (d) 6.

the existence of two Kondo temperature $T_{1K} = 2\tilde{\Gamma}$ and $T_{2K} = \tilde{\Delta} = \delta\tilde{V}^2/\tilde{\Gamma}$, associated to each Kondo regime.

$$\rho(\omega) \approx \frac{1}{\pi}\frac{2\tilde{\Gamma}}{\omega^2 + 4\tilde{\Gamma}^2} + \frac{1}{\pi}\frac{\tilde{\Delta}}{\omega^2 + \tilde{\Delta}^2} \,. \tag{16}$$

The above behavior of the DOS is due to quantum interference taking place in the transmission through the two different discrete states (the two quantum-dot levels) coupled to common leads. This phenomenon resembles the Dicke effect in optics, which takes place in the spontaneous emission of a pair of atoms radiating a photon with a wave length much larger than the separation between them.[16] The luminescence spectrum is characterized by a narrow and a broad peak, associated with long and short-lived states, respectively. The former state, weakly coupled to the electromagnetic field, is called *subradiant*, and the latter, strongly coupled, *superradiant* state. In the present case this effect is due to the indirect coupling between up-down QDs through the QW. The states strongly coupled to the QW yield an effective width $2\tilde{\Gamma}$ while those weakly coupled to the QW give a Dicke state with width $\tilde{\Delta}$. A similar result was found for a parallel double quantum dot without electron-electron interaction.[20]

The current and the differential conductance dJ/dV are two significant and experimentally measured quantities, which have been calculated numerically at finite source-drain biases.

Figure 4 displays the characteristic $J - V$ (solid line) and the differential conductance dJ/dV-V(dashed line) for two values of δV. For $\delta V = 0.1\Gamma$ the current shows a pronounced plateau around zero bias while for $\delta V = 0.5\Gamma$ the plateau is less defined. However in both cases the differential conductance shows an anomaly at zero bias.

Figure 5 shows details of the current and differential conductance around zero bias.

We can obtain the expressions for the current and the differential conductance by integrating over ω the transmission probability given in Eq. (15).

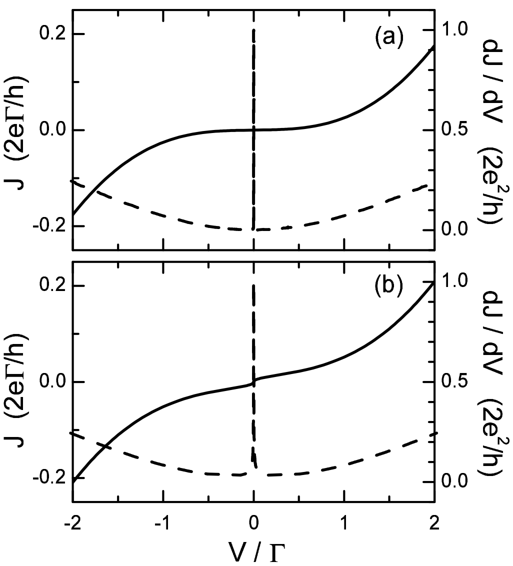

FIG. 4: Current (solid line) and differential conductance (dashed line) for $V_g = -3$, on site energy,$U = 6$ for a) $\delta V = 0.1$ and b) $\delta V = 0.5$

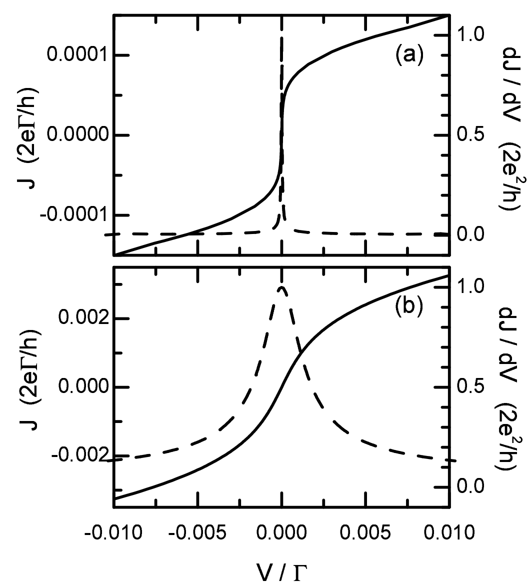

FIG. 5: Current (solid line) and Differential conductance (dashed line) for $V_g = -3$, on site energy, $U = 6$ for a) $\delta V = 0.1$ and b)$\delta V = 0.5$

$$\begin{aligned} J &\approx \frac{2e}{h}\left[V - 2\tilde{\Gamma}\arctan\left(\frac{V}{2\tilde{\Gamma}}\right) + \tilde{\Delta}\arctan\left(\frac{V}{\tilde{\Delta}}\right)\right], \\ \frac{\partial J}{\partial V} &\approx \frac{2e^2}{h}\left[1 - \frac{4\tilde{\Gamma}^2}{\left(\frac{V}{2}\right)^2 + 4\tilde{\Gamma}^2} + \frac{\tilde{\Delta}^2}{\left(\frac{V}{2}\right)^2 + \tilde{\Delta}^2}\right], \end{aligned} \tag{17}$$

We identify each term of the above equation as follows.

The first term in the right side of the Eqs.(17) is the contribution arising from an ideal unidimensional conductor. The second term comes from the Kondo-Fano state with temperature T_{1k} giving a quasi plateau for the current and almost zero differential conductance when $|V| \ll \tilde{\Gamma}$. The third term arises from the Kondo-Dicke state weakly coupled to the wire. It is responsible for an abrupt increase of the current and an amplification on the differential conductance around zero bias. Finally, for $|V| > \tilde{\Gamma}$, Kondo effect disappears.

IV. SUMMARY

We have studied the transport through two single side-coupled quantum dots using the finite-U slave boson mean field approach at $T = 0$. We have found that the transmission spectrum shows a structure with two anti-resonances localized at the renormalized energies of the quantum dots. The DOS of the system shows that when the Kondo correlations are dominant there are two Kondo regimes each with its own Kondo temperature. The above behavior of the DOS is due to quantum interference in the transmission through the two different resonance states of the quantum dots coupled to common leads. This result is analogous to the Dicke effect in optics. These phenomena have been analyzed as a function of the relevant parameters of the system.

Acknowledgments

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